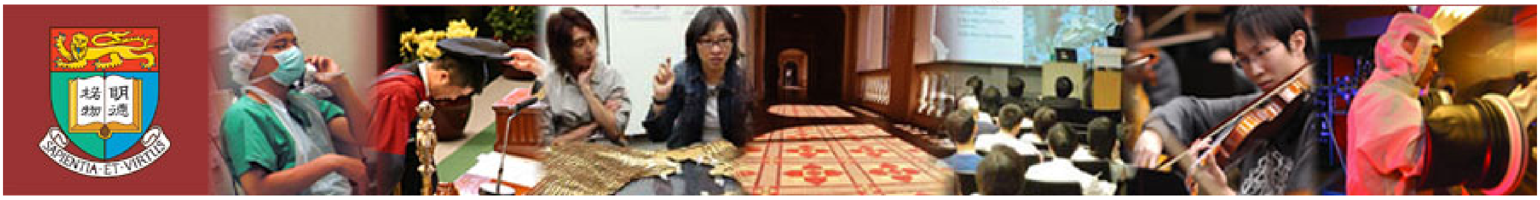

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A Novel Method for Distributed Generation and Capacitor Optimal Placement considering Voltage Profiles

Minnan Wang, *Student Member, IEEE,* and Jin Zhong, *Member, IEEE*

***Abstract*--To ensure the quality of power supply in distribution systems, bus voltages should be maintained within limits. Shunt capacitor banks installed along distribution feeders can reduce voltage drops. Moreover, distributed generations (DGs) can improve system voltage profile as well as supply real power. The locations of DGs and capacitors play an important role in maintaining voltage profiles.**

In this paper, two optimization models are proposed to obtain the optimal placements of DGs and capacitor banks to maintain better voltage profiles in distribution systems. First, the optimal DG placement problem is formulated as a modified optimal power flow (OPF) problem, with an innovative mathematic representation of voltage profile optimization. Then the capacitor optimal placement problem is modeled and solved. Both models are tested on the IEEE 41 bus distribution system, which is a radial system. Discussions are provided based on the results of case studies.

***Index Terms*-- Optimization, DG placement, capacitor placement, voltage profile, reactive power support.**

I. NOMENCLATURE

$P_{G,i}$	Active power output of generator located at bus i.
$P_{D,i}$	Active power demand at bus i.
$Q_{G,i}$	Reactive power output of generator located at bus i.
$Q_{D,i}$	Reactive power demand at bus i.
$Q_{C,i}$	Reactive power compensation provided by capacitor banks located at bus i.
U_i	Voltage magnitude of bus i.
δ_i	Voltage angle of bus i.
G_{ij}	Conductance of the line between bus i and bus j.
B_{ij}	Susceptance of the line between bus i and bus j.
U^L	Lowest bus voltage in the distribution system.
T_C	Total number of capacitor banks to be installed.
T_g	Total number of DG units to be installed.

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M. Wang and J. Zhong are with the Department of Electrical and Electronic Engineering, the University of Hong Kong, HKSAR, China. (E-mails: mnwang@eee.hku.hk; jzhong@eee.hku.hk). M. Wang is currently with the North China Electric Power Research Institute (NCEPRI), Beijing, China.

II. INTRODUCTION

IN a power system, the system operator is obligated to maintain voltage level of each customer bus within the required limit. To ensure voltage profiles are satisfactory in distribution systems, different standards have been established to provide stipulations or recommendations. For example, the American National Standards Institute (ANSI) standard C 84.1 has stipulated that voltage variations in a distribution system should be controlled within the range of -13% to 7% [1]. Actually in practice, many electricity companies try to control voltage variations within the range of ±5%.

One of the most widely adopted methods for improving voltage profiles of distribution systems is installing shunt capacitor banks along feeders. Since distributed generation (DG) units are introduced in distribution systems, DGs are also used for voltage supports. Shunt capacitor banks and DG units improve voltage profiles by changing power flow patterns. The locations of capacitor banks and DGs would have a significant impact on the effect of voltage profile enhancement.

In the past decade, much effort has been contributed to solving the optimal capacitor placement problem, utilizing different algorithms and considering different objectives. The capacitor placement problem could naturally be formulated as a mixed integer optimization problem. Various algorithms are used to solve the problem. For example, heuristic constructive algorithm (HCA) is presented in [2] in which the integer variables are represented by sigmoid function. Another heuristic method is adopted in [3] to obtain a near optimal solution for realistic sized systems, with an objective of minimizing harmonic levels, losses and capacitance costs. This method is expanded to take unbalanced load into consideration in [4]. Ant colony search algorithm (ACSA) is used in [5] to study the optimal capacitor placement problem in the distribution system as well as the optimal feeder reconfiguration problem.

Various objectives are proposed for optimal capacitor placement problems. The optimal capacitor placement and sizing problem in [6] has an objective function of minimizing the economic cost subject to voltage limits, sizes of installed capacitors at each bus, and power quality limits of harmonics. The impacts of capacitor placement on distribution system

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reliability are considered in [7] by defining two objective functions. The first one is the sum of reliability cost and investment cost, and the second one is the sum of reliability cost, cost of losses and investment cost.

On the other hand, considerable amount of research have been done on deciding optimal locations of DG units as well. An analytical method is proposed in [8] to decide the optimal locations of DGs along feeders, in radial as well as in networked systems, to minimize power losses. Optimal DG placement in [9] is obtained using exhaustive search to optimize system reliability and efficiency. System reliability is represented by SAIDI and efficiency by total loss. An iteration algorithm is used in [10], in which continuation power flow is adopted to decide the most sensitive bus to voltage collapse or maximum loading for DG installation. Objective functions considered in this work include reducing power loss, increasing power transfer capacity and maximum loading, and increasing voltage stability margins. Moreover, genetic algorithm is adopted in [11], to decide optimal DG placement with different load models. The objective function is based on a multi-objective index, which combines real and reactive power loss, voltage profile and MVA capacity. An adaptive weight particle swarm optimization (APSO) is proposed in [12] to optimize DG placement with the goal of minimizing real power losses within acceptable voltage limits. An Immune Algorithm (IA) based optimization approach is adopted in [13] to optimize voltage profiles by changing DG placements, while bus voltage limits and line current limits are handled as constraints. Genetic algorithm (GA) is used in [14] aiming to find the best balance between the cost and benefit of DG placements, in which benefit is represented by system reliability improvement.

In the literatures, very few papers use the optimization of voltage profile as objective functions. In this work, a novel method is proposed to decide the optimal locations of DGs and shunt capacitors to obtain an overall better voltage profile for a distribution system. The objective function is to maximize the lowest voltage level of the system to reach a better voltage profile. The locations of DGs and capacitors are formulated by binary variables as decision variables in the constraints.

This paper is structured as follows. Section III introduces the proposed mathematic model for optimal placement of DG and Section IV describes the model for optimal capacitor placement. In Section V, results of case studies on the IEEE 41 bus distribution system are presented, and discussions on the results are given. Finally conclusions drawn from this work are shown in Section VI.

III. Mathematic Model for Optimal Distributed Generation Placement

DGs introduced in distribution systems can provide active power as well as reactive power to local customers, and therefore reduce voltage drop in a radial distribution system. Placing DGs at proper locations could improve the voltage profile of a distribution system.

However, there are few clear criteria to evaluate how well a voltage profile is. It is hard to write a mathematic formulation of voltage profile for optimization. In this Section, we formulate the goodness of a voltage profile in an optimization model; hence the optimal locations of DGs could be obtained for a better voltage profile.

A. Objective Function

The most critical issue in optimizing voltage profile of a distribution system is to ensure the lowest bus voltage is above voltage lower limit. In a radial distribution system, bus voltage normally drops along the line.

The objective function in this model is to maximize the lowest bus voltage magnitude in the studied system, which is written as

$$\max \quad U^L \tag{1}$$

Here, U^L is an unknown state variable, which is determined by voltage levels of all buses. U^L is subject to constraint (4), and it is obtained as the optimization result of the model (1)-(11). The combination of (1) and (4) in the proposed model will find the optimal result that the lowest voltage in the system is maximized. This is one of the main features of the proposed model.

B. Constraints

1) Power flow constraints:

$$P_{G,i} - P_{D,i} = U_i \sum_{j=1}^{N} U_j \left[G_{ij} \cos\left(\delta_i - \delta_j\right) + B_{ij} \sin\left(\delta_i - \delta_j\right) \right] \tag{2}$$

$$Q_{G,i} - Q_{D,i} = -U_i \sum_{j=1}^{N} U_j \left[B_{ij} \cos\left(\delta_i - \delta_j\right) - G_{ij} \sin\left(\delta_i - \delta_j\right) \right] \tag{3}$$

2) Voltage constraints:

$$U_i \geq U^L \tag{4}$$

$$U_i \leq U^{\max} \tag{5}$$

$$U^{\min} \leq U^L \tag{6}$$

It should be noted that U^L in (4) is a variable, and the objective in this formulation is to maximize this variable, hence obtain an overall better voltage profile.

3) Generation constraints:

In this model, the locations of DGs are to be decided. DG units are allowed to be installed at bus i ($i \in N, i \neq sub$) of the studied system, where subscript ‘sub’ refer to the bus of distribution substation. Therefore for bus i, there are two options: either install a DG unit or not. Binary variable W_i is introduced to describe the installation of DG on bus i. If a DG is installed at bus i, W_i =1; otherwise W_i =0. So we have

$$\begin{cases} W_i = 1, & \text{if DG is installed at bus } i \\ W_i = 0, & \text{otherwise} \end{cases} \qquad i \in N,\, i \neq sub$$

In this case, if a DG is installed at bus i, its active and reactive power outputs should be within generation limits; otherwise, they should be equal to zero. Mathematically, this constraint can be written as

$$W_i P_{G,i}^{\min} \leq P_{G,i} \leq W_i P_{G,i}^{\max} \qquad i \in N, i \neq sub \tag{7}$$

$$W_i Q_{G,i}^{\min} \leq Q_{G,i} \leq W_i Q_{G,i}^{\max} \qquad i \in N, i \neq sub \tag{8}$$

where i is not the bus of distribution substation bus.

The actual real and reactive power injections from the substation bus should be within limits

$$P_{sub}^{\min} \le P_{G,sub} \le P_{sub}^{\max} \tag{9}$$

$$Q_{sub}^{\min} \le Q_{G,sub} \le Q_{sub}^{\max} \tag{10}$$

4) Generator number constraint: the total number of installed DGs could be equal to a predetermined number, which can be represented as

$$\sum_{i=1, i \ne sub}^{N} W_i = T_g \tag{11}$$

IV. Mathematic Model for Optimal Capacitor Placement

The optimal capacitor placement model can be formulated using the same objective function (1) as presented in Section III

$$\max \quad U^L \tag{1}$$

subject to following constraints.

1) Power flow constraints:

$$P_{G,i} - P_{D,i} = U_i \sum_{j=1}^{N} U_j \left[G_{ij} \cos\left(\delta_i - \delta_j\right) + B_{ij} \sin\left(\delta_i - \delta_j\right) \right] \tag{12}$$

$$\begin{aligned} Q_{G,i} - Q_{D,i} + Q_{C,i} \\ = -U_i \sum_{j=1}^{N} U_j \left[B_{ij} \cos\left(\delta_i - \delta_j\right) - G_{ij} \sin\left(\delta_i - \delta_j\right) \right] \end{aligned} \tag{13}$$

In this model, we assume that the locations of DGs are already decided.

2) Voltage limit constraints (4) (5) (6).

3) Capacitor installation constraints:

Here, we assume that each capacitor bank is of a fixed reactive power capacity Q_0.

Integer variable X_i is introduced to represent the number of capacitor banks installed at bus i. The reactive power of shunt capacitor banks at bus i is given by

$$Q_{C,i} = X_i Q_0 \tag{14}$$

In this paper, we assume the total number of capacitor banks in the whole system is predetermined as T_C, which is represented by

$$\sum_{i=1}^{N} X_i = T_C \tag{15}$$

$$X_i \ge 0, \ X_i \text{ is integer variable} \tag{16}$$

Constraints (14) and (15) represent the optimal assignment of T_C capacitor banks on different bus bars. The constraints could be modified to represent other assumptions, for example, the total number of capacitors is lower than a predetermined number, or certain buses must have capacitors installed, etc.

4) Power generation limits

$$P_{G,i}^{\min} \le P_{G,i} \le P_{G,i}^{\max} \tag{17}$$

$$Q_{G,i}^{\min} \le Q_{G,i} \le Q_{G,i}^{\max} \tag{18}$$

V. Case Study and Discussions

The mathematic models proposed in Section III and IV are tested with the IEEE 41 bus distribution system. This system is modified based on the system described in [15]. The structure of the system is shown in Fig.1. Bus 1 is the substation bus. The parameters of the system are given in the Appendix. The total active and reactive loads are 4635 kW and 3250 kVar, respectively. Table I shows the initial voltage profile of the system without installing any DG unit or shunt capacitor. It can be seen that voltage drops along the feeders and the lowest bus voltage is 0.827 p.u., appearing at buses 40 and 41. The lowest voltage buses are highlighted with grey color in the table.

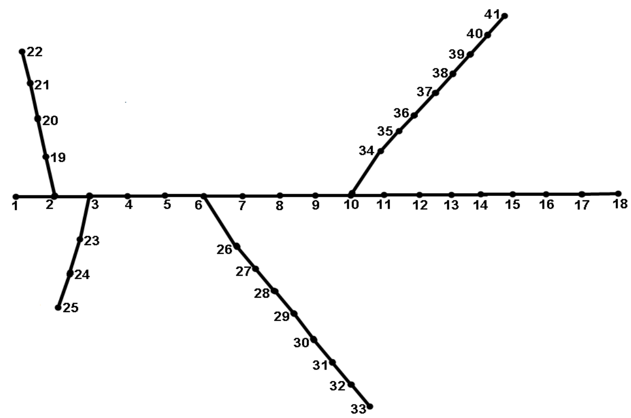

Fig. 1. Structure of the IEEE 41 bus distribution system

TABLE I
INITIAL BUS VOLTAGES OF STUDIED SYSTEM

Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)
1	1.000	12	0.861	23	0.972	34	0.862
2	0.996	13	0.855	24	0.966	35	0.859
3	0.976	14	0.852	25	0.962	36	0.846
4	0.964	15	0.851	26	0.921	37	0.837
5	0.953	16	0.849	27	0.918	38	0.833
6	0.923	17	0.847	28	0.906	39	0.828
7	0.913	18	0.846	29	0.898	40	0.827
8	0.901	19	0.995	30	0.894	41	0.827
9	0.882	20	0.992	31	0.890		
10	0.864	21	0.991	32	0.889		
11	0.863	22	0.990	33	0.888		

A. Optimal DG Placement in Normal Operation Mode

In Case I, the active and reactive power supply from the substation is assumed to be large enough to support all of the loads in the system. The total number of DG units to be installed in the system is set to one, with a rated active power capacity of 1500 kW, which is 32% of the total active power demand, and a rated reactive power capacity of 1200 kVar.

The optimization results show that the optimal DG placement is at bus 38. The actual active and reactive power outputs of the DG unit are 1496 kW and 1200 kVar, respectively, which is very close to its rated capacity. The voltage profile of the studied system in this case is given in Table II. It could be seen that the lowest bus voltage is 0.928

p.u. at buses 32 and 33.

TABLE II
CASE I - BUS VOLTAGES WHEN SUBSTATION CONNECTED AND DG OPTIMAL LOCATION AT BUS 38

Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)
1	1.000	12	0.953	23	0.982	34	0.956
2	0.997	13	0.947	24	0.975	35	0.957
3	0.986	14	0.945	25	0.972	36	0.964
4	0.980	15	0.943	26	0.958	37	0.969
5	0.974	16	0.942	27	0.956	38	0.973
6	0.960	17	0.940	28	0.945	39	0.969
7	0.959	18	0.939	29	0.936	40	0.968
8	0.957	19	0.997	30	0.933	41	0.968
9	0.956	20	0.993	31	0.929		
10	0.955	21	0.992	32	0.928		
11	0.954	22	0.992	33	0.928		

In Case II, the active and reactive power capacities of the DG unit are increased to 5000 kW and 4000 kVar, respectively, which are enough to support the total load in the system. The optimization results show that the optimal location of the DG unit is at bus 9, and its actual active power output is 4360 kW. The active power output of the substation is 608 kW. Table III shows the system voltage profile of case II. The lowest bus voltage here is 0.951 p.u., appearing at buses 32, 33, 40, and 41.

TABLE III
CASE – II BUS VOLTAGES WHEN SUBSTATION CONNECTED AND DG OPTIMAL LOCATION AT BUS 9

Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)
1	0.998	12	0.981	23	0.987	34	0.982
2	0.996	13	0.976	24	0.981	35	0.979
3	0.991	14	0.974	25	0.977	36	0.968
4	0.990	15	0.972	26	0.982	37	0.960
5	0.989	16	0.971	27	0.979	38	0.957
6	0.984	17	0.969	28	0.968	39	0.953
7	0.980	18	0.968	29	0.960	40	0.952
8	0.989	19	0.996	30	0.957	41	0.952
9	1.000	20	0.992	31	0.953		
10	0.984	21	0.992	32	0.952		
11	0.983	22	0.991	33	0.952		

Comparing Case I and Case II, it can be seen that the optimal location of DG unit varies with its capacity. With different sized DG, the optimal location could be different.

B. Optimal DG Placement in Islanding Operation Mode

When a distribution system suffers from power outage and is disconnected from its upstream substation, some types of DGs have the capability of supplying part of the de-energized area in the islanding operation mode. Here, we use the proposed model to decide the optimal locations of DG units. In Case III, the system is de-energized from bus 1. The total number of DG units to be installed in this study case is still set to one. The rated active and reactive power capacities of the DG are given, which are 5000 kW and 4000 kVar respectively, enough to supply the total load of the system.

Using the optimal DG placement model proposed in Section III to consider the DG location under islanding operation mode, we found that the optimal DG placement is at bus 8, which is a central location of the studied system. The active power output of the DG is 4880 kW, and reactive power output is 3448 kVar. The voltage profile when the DG is placed at its optimal location, bus 8, is shown in Table IV. It could be seen from the results that the lowest bus voltage is 0.934 p.u., appears bus 40 and bus 41.

TABLE IV
CASE III - BUS VOLTAGES OF ISLAND OPERATION AND DG OPTIMAL LOCATION AT BUS 8

Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)
1	0.949	12	0.966	23	0.947	34	0.964
2	0.949	13	0.958	24	0.940	35	0.962
3	0.951	14	0.956	25	0.937	36	0.951
4	0.955	15	0.955	26	0.971	37	0.943
5	0.960	16	0.953	27	0.969	38	0.939
6	0.973	17	0.951	28	0.958	39	0.935
7	0.984	18	0.951	29	0.950	40	0.934
8	1.000	19	0.948	30	0.946	41	0.934
9	0.983	20	0.945	31	0.942		
10	0.966	21	0.944	32	0.941		
11	0.966	22	0.943	33	0.941		

Compare optimal DG placement results of the grid-connected operation (Case II, Table III) and the island operation (Case III, Table IV), we can find that the optimal locations are different. The comparison is given in Table V.

TABLE V
COMPARISON OF CASE II AND CASE III

Operation mode	Optimal DG location	DG output (MW)	Substation output (MW)	U^L @ buses
Grid-connected	bus 9	4360	608	0.951 @ 32, 33, 40, 41
Island operation	bus 8	4880	--	0.934 @ 40, 41

From Table III - Table V, we can see that the voltage profile is better in grid-connected operation mode than island operation mode. And the DG optimal locations could be different considering different operation modes.

In Case IV, we test a case if the DG is arbitrarily located, for example at bus 9. The system voltage profile of Case IV is shown in Table VI. The lowest bus voltage is 0.907, appearing at bus 33.

Comparing the results of Table IV (optimal DG location) and Table VI (arbitrary DG location), the impacts of DG locations on system voltage profile can be clearly seen. The lowest bus voltage improves from 0.903 p.u. to 0.934 p.u. when the DG location moves from an arbitrary location bus 9 to optimal location bus 8, although bus 9 and bus 8 are very close to each other.

TABLE VI
CASE IV - BUS VOLTAGES OF ISLAND OPERATION AND DG ARBITRARILY LOCATED AT BUS 9

Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)	Bus Index	Bus Voltage (p.u.)
1	0.916	12	0.981	23	0.914	34	0.982
2	0.916	13	0.976	24	0.907	35	0.979
3	0.918	14	0.973	25	0.903	36	0.968
4	0.922	15	0.972	26	0.939	37	0.960
5	0.927	16	0.971	27	0.936	38	0.957
6	0.941	17	0.969	28	0.925	39	0.953
7	0.952	18	0.968	29	0.916	40	0.952
8	0.969	19	0.915	30	0.913	41	0.952
9	1.000	20	0.911	31	0.909		
10	0.984	21	0.910	32	0.908		
11	0.983	22	0.910	33	0.907		

C. Optimal Capacitor Placement

In this sub-section, the optimal placement of capacitors is decided using the model proposed in Section VI. The predetermined total capacitor bank number, T_C varies from 2 to 8 in following cases, and the capacity of each capacitor bank, Q_0 is fixed at 400 kVar.

In Case V, the system is grid-connected. And in Case VI, the system is in islanding operation and is supplied by a DG unit located at bus 9. Still, the rated active and the reactive power of the DG unit are 5000 kW and 4000 kVar, respectively.

The detailed results indicating the optimal locations and numbers of shunt capacitor banks are shown in Table VII and Table VIII, respectively. The capacitor banks are optimally assigned to different buses to maintain better voltage profiles.

TABLE VII
CASE V - OPTIMAL CAPACITOR PLACEMENT IN GRID-CONNECTED MODE

Total capacitor bank No.	Total capacitor capacity	Bank no.		Bank location	U^L @ buses
2	800	1	@	39	0.871 @ 38
		1	@	40	
4	1600	1	@	14	0.899 @ 18, 38
		1	@	38	
		1	@	39	
		1	@	40	
6	2400	1	@	17	0.918 @ 13,14,15,16,38,40,41
		1	@	30	
		2	@	38	
		2	@	39	
8	3200	1	@	12	0.932 @ 13,14,15,16,37,38,41
		1	@	17	
		2	@	30	
		2	@	38	
		1	@	39	
		1	@	40	

TABLE VIII
CASE VI - OPTIMAL CAPACITOR PLACEMENT IN ISLAND OPERATION MODE

Total capacitor bank No.	Total capacitor capacity	Bank no.		Bank location	U^L @ buses
2	800	1	@	25	0.920 @ 25
		1	@	30	
4	1600	1	@	2	0.936 @ 25, 31, 32
		1	@	23	
		1	@	25	
		1	@	33	
6	2400	1	@	20	0.951 @ 24, 25, 30
		1	@	23	
		2	@	25	
		1	@	31	
		1	@	32	
8	3200	1	@	21	0.957 @ 24, 25, 40, 41
		1	@	24	
		2	@	25	
		2	@	29	
		1	@	33	
		1	@	34	

Comparing Table VII and Table VIII with the lowest voltages in Table I and Table VI, respectively, it can be seen that the system voltage profile is largely improved due to capacitor placements. With the increasing number of capacitor banks, the lowest bus voltage increases monotonously. Moreover, it can be seen from the results that installing capacitor banks at several adjacent buses is likely to provide a better system voltage profile than installing them all together at one single bus.

VI. CONCLUSIONS

In this paper, methods for optimal DG unit and capacitor placements in distribution systems are studied. Two innovative mathematic models are proposed to solve the problems respectively, both of which are formulated as modified OPF problems.

The effectiveness of the proposed models is tested using the IEEE 41 bus system. It can be concluded from the case studies that the strategic placement of DG units would have a strong influence on the voltage profile improvement of the distribution system. Moreover, capacitor banks could be assigned optimally to procure a better voltage profile.

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VIII. Appendix

A. Parameters of the IEEE 41-bus distribution systems

TABLE IX
PARAMETERS OF THE IEEE 41 BUS DISTRIBUTION SYSTEMS

Line Index	To Bus Index	From Bus Index	Line Resistance R (ohm)	Line Reactance X (ohm)	From bus Load Active Power Pl (kW)	From bus Load Reactive Power Ql (kvar)
1	1	2	0.0992	0.0470	100	60
2	2	3	0.4930	0.2511	90	40
3	3	4	0.3660	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.8190	0.7070	60	20
6	6	7	0.1872	0.6188	200	100
7	7	8	0.7114	0.2351	200	100
8	8	9	1.0300	0.7400	60	20
9	9	10	1.0440	0.7400	60	20
10	10	11	0.1966	0.0650	45	30
11	11	12	0.3744	0.1238	60	35
12	12	13	1.4680	1.1550	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.5910	0.5260	60	10
15	15	16	0.7463	0.5450	60	20
16	16	17	1.2890	1.7210	60	20
17	17	18	0.7320	0.5470	90	40
18	2	19	0.1640	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.8980	0.7091	420	200
24	24	25	0.8960	0.7011	420	200
25	6	26	0.2030	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.0590	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.9630	150	70
31	31	32	0.3105	0.3619	210	100
32	32	33	0.3410	0.5302	60	40
33	10	34	0.2030	0.1034	60	25
34	34	35	0.2842	0.1447	60	25
35	35	36	1.0590	0.9337	60	20
36	36	37	0.8042	0.7006	120	70
37	37	38	0.5075	0.2585	200	600
38	38	39	0.9744	0.9630	150	70
39	39	40	0.3105	0.3619	210	100
40	40	41	0.3410	0.5302	60	40

IX. Biographies

Minnan Wang (S'08) received her B.Sc degree from the Department of Electrical Engineering in Tsinghua University, Beijing, China, and MPhil degree from the Department of Electrical and Electronic Engineering in the University of Hong Kong, HKSAR, China, in 2008 and 2010, respectively. She is currently working at the North China Electric Power Research Institute, Beijing, China.

Jin Zhong (S'00, M'05) received her B.Sc. degree from Tsinghua University, Beijing, China, in 1995, the M. Sc. degree from China Electric Power Research Institute in 1998 and the Ph.D. degree from Chalmers University of Technology, Gothenburg, Sweden, in 2003. At present, she is an Associate Professor in the Department of Electrical and Electronic Engineering, the University of Hong Kong. Her areas of interest are electricity sector deregulation, ancillary service pricing, power system planning, and smart grid.